

MEET THE

4
FANTASTIC FOUR



CA FOUNDATION BATCH LAUNCH FOR JUNE 2024

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CODE : SS12



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INTRODUCTION

- The terms 'Probably' 'in all likelihood', 'chance', 'odds in favour', 'odds against' are too familiar nowadays and they have their origin in a branch of Mathematics, known as Probability.
- In recent time, probability has developed itself into a full-fledged subject and become an integral part of statistics.

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HISTORY

- It is rather surprising to know that the first application of probability was made by a group of mathematicians in Europe about three hundreds years back to enhance their chances of winning in different games of gambling.



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DIVISIONS OF PROBABILITY

SUBJECTIVE PROBABILITY

- Personal judgement
- Experience
- Influenced by the personal belief, attitude and bias of the person applying it

OBJECTIVE PROBABILITY

- Based on Rules and Maths



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RANDOM EXPERIMENT

- If all possible outcomes of an experiment are known but the exact output cannot be predicted in advance then the experiment is known as Random Experiment

For example if a coin is tossed, then we get two outcomes—

Head (H) and Tail (T).

It is impossible to say in advance whether a Head or a Tail would turn up when we toss the coin once.

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RANDOM EXPERIMENT

EXAMPLE

Rolling a dice (or any number of dice),

Drawing items from a box containing both defective and non-defective items,

Drawing cards from a pack of well shuffled fifty two cards etc. are all random experiments.



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SAMPLE SPACE



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- The set of all possible outcomes of an experiment is called the sample space.
- **Example:**
 - List the sample space in tossing a fair coin.
 $S = \{H, T\}$

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COINS

- Total number of elements in sample space while tossing a coin is given by 2^n
 1. If a coin is tossed once $2^1 = 2$
{H, T}
 1. If two coins are tossed once or one coin tossed twice $2^2 = 4$
{HH, HT, TH, TT}
 1. If three coins are tossed once or one coin is tossed thrice $2^3 = 8$
{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

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- Total number of elements in sample space while tossing

a dice is given by 6^n

1. If a dice is rolled once $6^1 = 6$

$\{1, 2, 3, 4, 5, 6\}$

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2. If two die is rolled once or one dice is rolled twice

$$6^2 = 36$$

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

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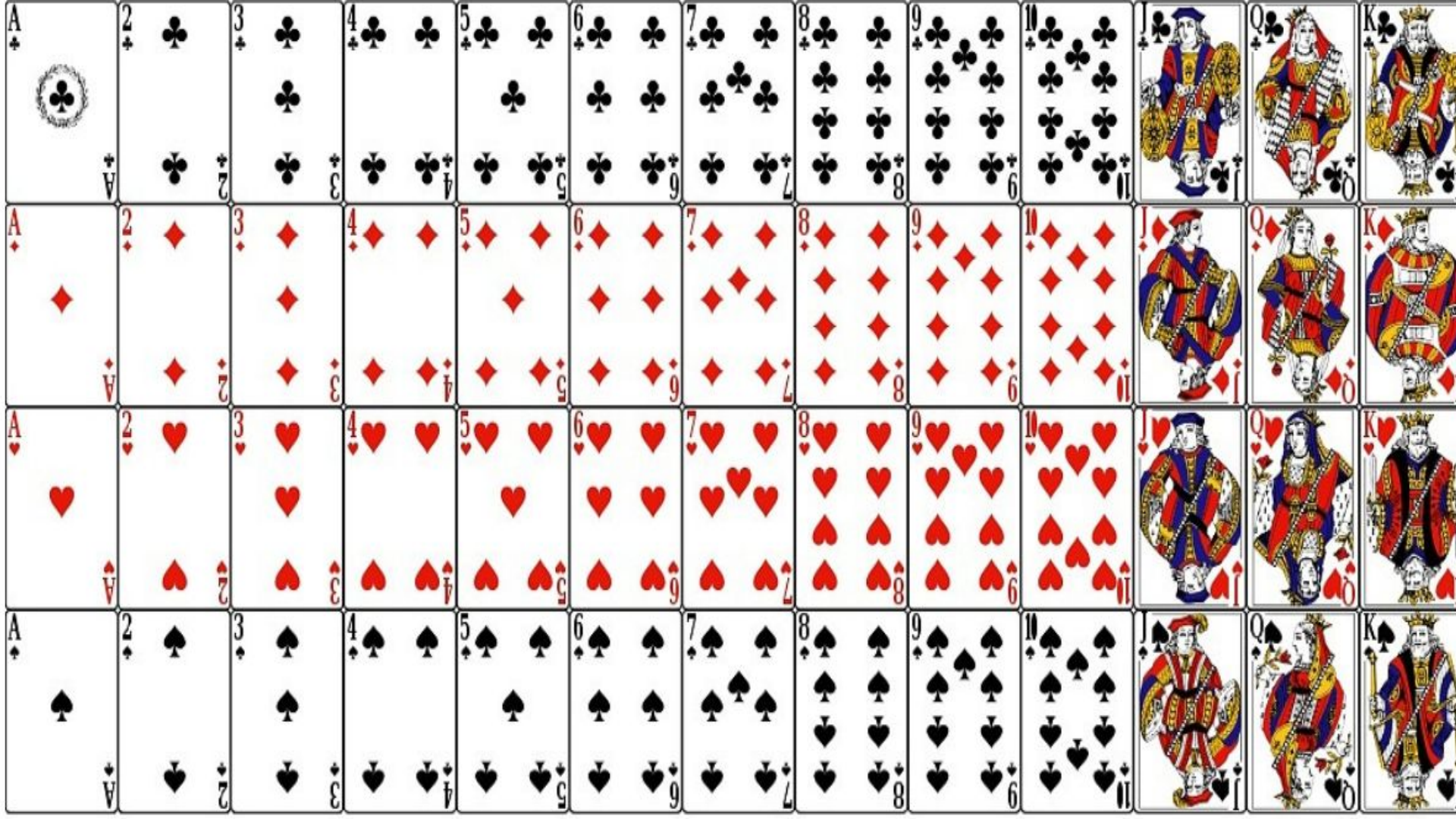


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3. If three dice are rolled once or one dice is rolled thrice

$$6^3 = 216$$

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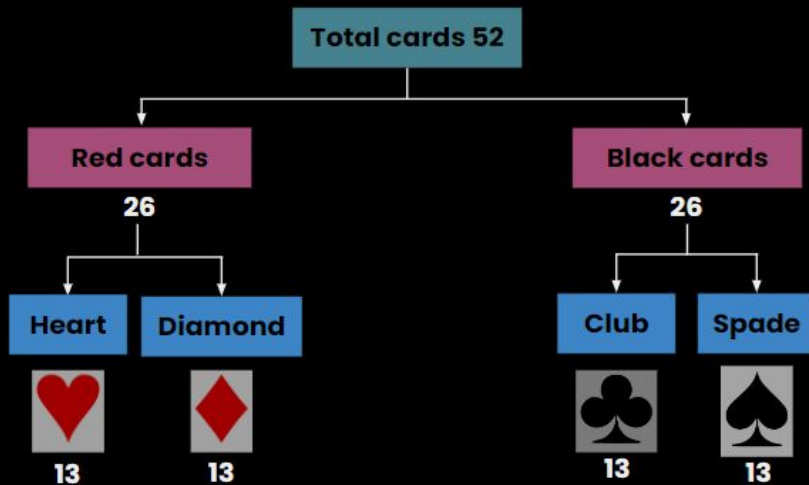
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CARDS



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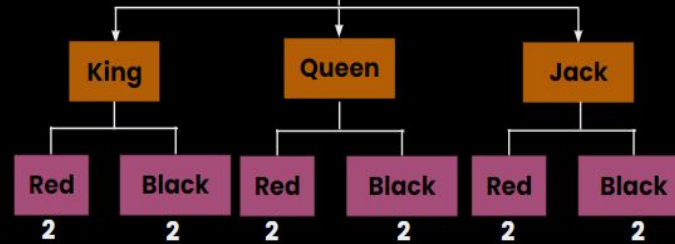
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CARDS



Total face cards $12 = 4 \times 3$



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EVENT

Any subset of a sample space is called event.

- The results or outcomes of a random experiment are known as events

EXAMPLE

The Event of getting a Prime number in a single throw of a die

$$E = \{2, 3, 5\}$$



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SURE EVENT

- If probability of occurrence of an event is 1
- **Example**
Getting a number less than 7 on a throw of a single dice

IMPOSSIBLE EVENT

- If probability of occurrence of an event is 0
- **Example**
Getting a number 7 on a throw of a single dice



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SIMPLE EVENT

- Number of elements = 1

Example

Random Experiment :

Tossing two coins

A : getting both heads

COMPOSITE / COMPOUND EVENT

- Number of elements > 1

Example

Random Experiment :

Tossing two coins

B : getting atleast one head

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EQUALLY LIKELY EVENTS / MUTUALLY SYMMETRIC EVENTS / EQUI - PROBABLE EVENTS

- Events are equally likely if they have same probability of occurrence .

Example

- Getting head and getting tail on the toss of fair coins .



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MUTUALLY EXCLUSIVE / DISJOINT / INCOMPATIBLE EVENTS

- Two events A and B are said to be mutually exclusive or disjoint if their simultaneous occurrence is impossible .
- If A and B are mutually exclusive then $A \cap B = \Phi$.
- **EXAMPLE** : Random Experiment : Throwing a dice

A : getting odd number

B : getting even number

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EXHAUSTIVE EVENTS

- The events A_1, A_2, A_3, \dots are known to form an exhaustive set if one of these events must necessarily occur.
- **Example** the two events Head and Tail, when a coin is tossed once, are exhaustive as no other event except these two can occur.
- Events whose union is equal to sample space .



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CLASSICAL DEFINITION OF PROBABILITY

Also called Prior definition of Probability

This formula is Event based

It is given by Bernoulli and Laplace

$$P(A) = \frac{n_A}{n} = \frac{\text{No. of equally likely events favourable to A}}{\text{Total no. of equally likely events}}$$



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RESULTS



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(a) The probability of an event lies between 0 and 1, both inclusive.

i.e. $0 \leq P(A) \leq 1$

..... (15.3)

When $P(A) = 0$, A is known to be an impossible event and when $P(A) = 1$, A is known to be a sure event.

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COMPLEMENTARY PROBABILITY

Probability of non - occurrence of an event A is denoted by $P(A')$ or $P(\overline{A})$ is called as complimentary event of A

$$P(A) + P(A') = 1$$



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ODDS IN FAVOUR



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Odds in favour of an event A

$$= \frac{\text{no of favorable events to A}}{\text{no of unfavorable events to A}}$$

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ODDS AGAINST AN EVENT



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Odds against an event A

$$= \frac{\text{no of unfavourable events to A}}{\text{no of favourable events to A}}$$

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PROBABILITY AN EVENT



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$$P(A) = \frac{\text{no of favourable events to A}}{\text{no of favourable + no of unfavourable}}$$

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LIMITATIONS OF CLASSICAL PROBABILITY

- i. It is applicable only when the total no. of events is finite.
- ii. It can be used only when the events are equally likely or equi-probable.
- iii. This definition has only a limited field of application like coin tossing, dice throwing, drawing cards etc. where the possible events are known well in advance



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Example A coin is tossed three times. What is the probability of getting:

- (i) 2 heads
- (ii) at least 2 heads.



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Example A dice is rolled twice. What is the probability of getting a difference of 2 points?



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Example Two dice are thrown simultaneously. Find the probability that the sum of points on the two dice would be 7 or more.



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Example What is the chance of picking a spade or an ace not of spade from a pack of 52 cards?



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Example A committee of 7 members is to be formed from a group comprising 8 gentlemen and 5 ladies. What is the probability that the committee would comprise:

- (a) 2 ladies,
- (b) at least 2 ladies.



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Example Find the probability that a four digit number comprising the digits 2, 5, 6 and 7 would be divisible by 4.



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Write down the correct answer. Each question carries 1 mark.

Que. 46 Probability of getting a head when two unbiased coins are tossed simultaneously is

- (a) 0.25
- (b) 0.50
- (c) 0.20
- (d) 0.75



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Write down the correct answer. Each question carries 1 mark.



Que. 47 If an unbiased coin is tossed twice, the probability of obtaining at least one tail is

- (a) 0.25
- (b) 0.50
- (c) 0.75
- (d) 1.00

CODE : SS12

Write down the correct answer. Each question carries 1 mark.

Que. 48 If an unbiased die is rolled once, the odds in favour of getting a point which is a multiple of 3 is

- (a) 1:2
- (b) 2:1
- (c) 1:3
- (d) 3:1



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Que. 49 A bag contains 15 one rupee coins, 25 two rupee coins and 10 five rupee coins. If a coin is selected at random from the bag, then the probability of not selecting a one rupee coin is

- (a) 0.30
- (b) 0.70
- (c) 0.25
- (d) 0.20



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Write down the correct answer. Each question carries 1 mark.



Que. 51 If two letters are taken at random from the word HOME, what is the Probability that none of the letters would be vowels?

(a) $1/6$

(b) $1/2$

(c) $1/3$

(d) $1/4$

CODE : SS12 Write down the correct answer. Each question carries 2 mark.

Que. 2 What is the chance of throwing at least 7 in a single cast with 2 dice?

- (a) $5/12$
- (b) $7/12$
- (c) $1/4$
- (d) $17/36$



CODE : SS12 Write down the correct answer. Each question carries 2 mark.

Que. 3 What is the chance of getting at least one defective item if 3 items are drawn randomly from a lot containing 6 items of which 2 are defective item?

- (a) 0.30
- (b) 0.20
- (c) 0.80
- (d) 0.50



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Write down the correct answer. Each question carries 2 mark.

Que. 4 If two unbiased dice are rolled together, what is the probability of getting no difference of points?

- (a) $1/2$
- (b) $1/3$
- (c) $1/5$
- (d) $1/6$



CODE : SS12 *Write down the correct answer. Each question carries 2 mark.*



Que. 6 There are 10 balls numbered from 1 to 10 in a box. If one of them is selected at random, what is the probability that the number printed on the ball would be an odd number greater than 4?

- (a) 0.50
- (b) 0.40
- (c) 0.60
- (d) 0.30

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Write down the correct answer. Each question carries 2 mark.

Que. 7 Following are the wages of 8 workers in rupees:

50, 62, 40, 70, 45, 56, 32, 45

If one of the workers is selected at random, what is the probability that his wage would be lower than the average wage?

- (a) 0.625
- (b) 0.500
- (c) 0.375
- (d) 0.450



CODE : SS12 Write down the correct answer. Each question carries 5 mark.

Que.1 What is the probability that a leap year selected at random would contain 53 Saturdays?

(a) $1/7$

(b) $2/7$

(c) $1/12$

(d) $1/4$



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Write down the correct answer. Each question carries 5 mark.

Que. 2 If an unbiased coin is tossed three times, what is the probability of getting more than one head?

(a) $1/8$

(b) $3/8$

(c) $1/2$

(d) $1/3$



CODE : SS12 Write down the correct answer. Each question carries 5 mark.

Que. 3 If two unbiased dice are rolled, what is the probability of getting points neither 6 nor 9?

- (a) 0.25
- (b) 0.50
- (c) 0.75
- (d) 0.80



CODE : SS12**Write down the correct answer. Each question carries 5 mark.****Que. 4** What is the probability that 4 children selected at random would have different birthdays?

(a) $\frac{364 \times 363 \times 362}{(365)^3}$

(b) $\frac{6 \times 5 \times 4}{7^3}$

(c) $1/365$

(d) $(1/7)^3$



CODE : SS12**Write down the correct answer. Each question carries 5 mark.**

Que. 15 A packet of 10 electronic components is known to include 2 defectives. If a sample of 4 components is selected at random from the packet, what is the probability that the sample does not contain more than 1 defective?

- (a) $1/3$
- (b) $2/3$
- (c) $13/15$
- (d) $3/15$

EXERCISE- Set (C)

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Que 19 Four digits 1, 2, 4 and 6 are selected at random to form a four digit number . What is the probability that the number so formed would be divisible by 4.



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CODE : SS12 *Write down the correct answer. Each question carries 1 mark.*

Que. 5 An event that can be split into further events is known as

- (a) Complex event
- (b) Mixed event
- (c) Simple event
- (d) Composite event



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Write down the correct answer. Each question carries 1 mark.



Que. 10 If an unbiased coin is tossed once, then the two events Head and Tail are

- (a) Mutually exclusive
- (b) Exhaustive
- (c) Equally likely
- (d) All these (a), (b) and (c).

CODE : SS12 Write down the correct answer. Each question carries 1 mark.

Que. 22 If $P(A) = \frac{5}{9}$, then the odds against the event A is

- (a) 5 : 9
- (b) 5 : 4
- (c) 4 : 5
- (d) 5 : 14



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**SET THEORETIC APPROACH
TO PROBABILITY**

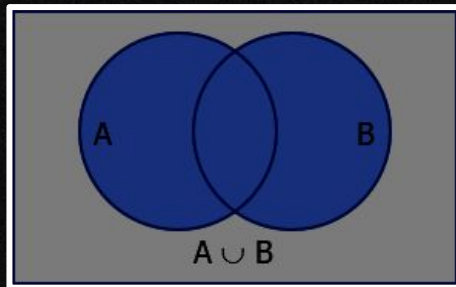
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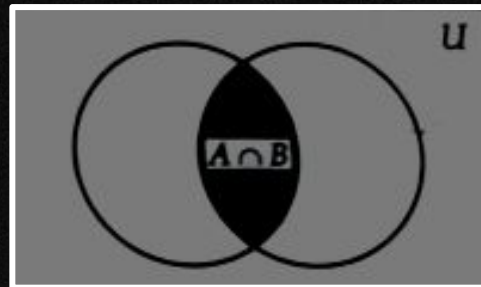
Venn Diagram

Union of Sets:



OR

Intersection of Sets:



AND

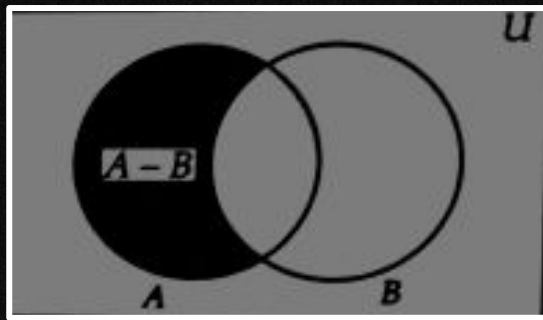
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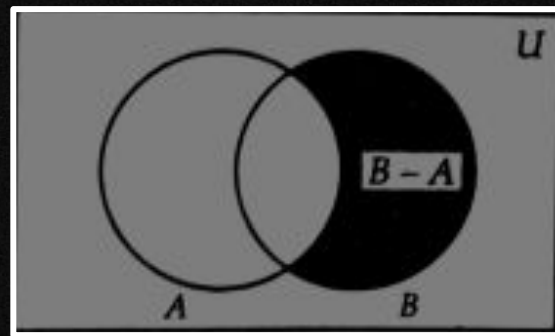
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Difference of Sets:



Only A



Only B

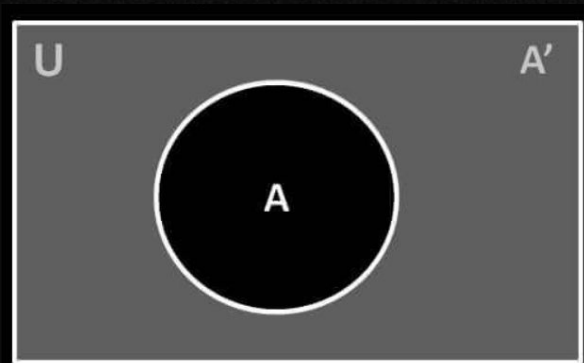
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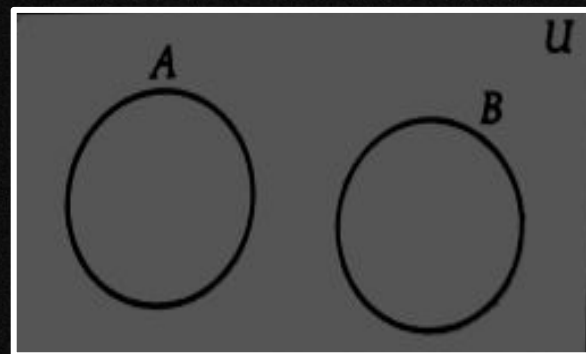
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Complement of sets



Disjoint of Sets:



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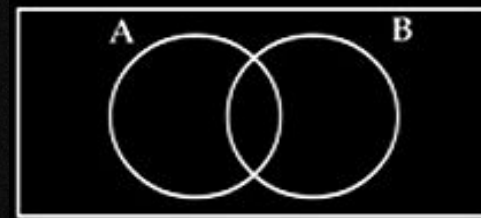
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Important Result on number of elements in set

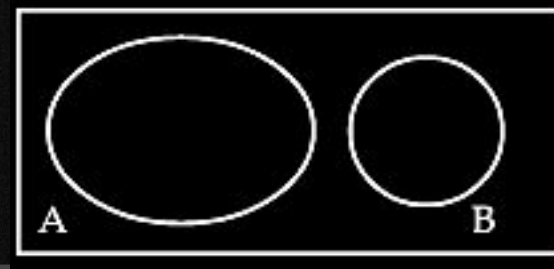
- Thus $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



- If A and B are disjoint sets, then

$$n(A \cup B) = n(A) + n(B)$$

$$\text{as } n(A \cap B) = 0$$



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Important Result on number of elements in set

- For three sets P, Q and R

$$n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$$

- When P, Q and R are disjoint sets

$$= n(P) + n(Q) + n(R)$$



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De Morgan's Law

- For any two sets A and B

(i) $(A \cup B)' = (A' \cap B')$

(ii) $(A \cap B)' = (A' \cup B')$



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SET THEORETIC APPROACH TO PROBABILITY

- A sample space may be defined as a non-empty set containing all the elementary events of a random experiment as sample points.
- A sample space is denoted by S or Ω .
- An event A may be defined as a non-empty subset of S .



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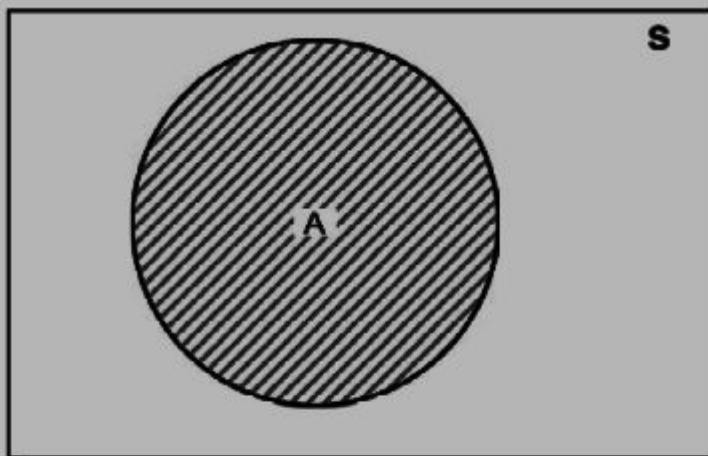


Figure 15.1

Showing an event A  and the sample space S

- **Universal Set – Sample space (S)**
- **Elements – Sample points**
- **Event Set A – Subset of S**

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SET THEORETIC APPROACH TO PROBABILITY

- Let us consider a sample space with a finite no. of sample points, $n(S)$. We assume that all these sample points are equally likely.
- If an event A which is a subset of S , contains $n(A)$ sample points,
- then the probability of A is defined as the ratio of the number of sample points in A to the total number of sample points in S . i.e.

$$P(A) = \frac{n(A)}{n(S)}$$



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IMPORTANT RESULTS

- $P(E) \geq 0$
- $P(\Phi) = 0$
- $P(S) = 1$
- If E_1 and E_2 are mutually exclusive events $P(E_1 \cap E_2) = 0$



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ADDITION THEOREM OF PROBABILITY

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RESULTS

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

- If A ,B and C are mutually exclusive

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$



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RESULTS

- Two events A and B are exhaustive if

$$P(A \cup B) = 1$$

- Three events A, B and C are exhaustive if

$$P(A \cup B \cup C) = 1$$

- Three events A, B and C are equally likely if

$$P(A) = P(B) = P(C)$$



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RESULTS

- Probability that only event A occurs

$$P(A-B) = P(A \cap B') = P(A) - P(A \cap B)$$

- Probability that only event B occurs

$$P(B-A) = P(B \cap A') = P(B) - P(A \cap B)$$



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Example Three events A, B and C are mutually exclusive, exhaustive and equally likely.

What is the probability of the complementary event of A?



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Example A number is selected from the first 25 natural numbers. What is the probability that it would be divisible by 4 or 7?



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Example A number is selected at random from the first 1000 natural numbers. What is the probability that it would be a multiple of 5 or 9?



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Example The probability that an Accountant's job applicant has a B. Com. Degree is 0.85, that he is a CA is 0.30 and that he is both B. Com. and CA is 0.25 out of 500 applicants, how many would be B. Com. or CA?



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Example There are three persons A, B and C having different ages. The probability that A survives another 5 years is 0.80, B survives another 5 years is 0.60 and C survives another 5 years is 0.50. The probabilities that A and B survive another 5 years is 0.46, B and C survive another 5 years is 0.32 and A and C survive another 5 years 0.48. The probability that all these three persons survive another 5 years is 0.26. Find the probability that at least one of them survives another 5 years.

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Example If $P(A-B) = 1/5$, $P(A) = 1/3$ and $P(B) = 1/2$, what is the probability that out of the two events A and B, only B would occur?



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SET A

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Write down the correct answer. Each question carries 1 mark.

Que. 8 If $P(A \cap B) = 0$, then the two events A and B are

- (a) Mutually exclusive
- (b) Exhaustive
- (c) Equally likely
- (d) Independent.



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SET A

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Write down the correct answer. Each question carries 1 mark.

Que. 9 If for two events A and B, $P(A \cup B) = 1$, then A and B are

- (a) Mutually exclusive events
- (b) Equally likely events
- (c) Exhaustive events
- (d) Dependent events.



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SET A

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Write down the correct answer. Each question carries 1 mark.

Que. 23 If A, B and C are mutually exclusive and exhaustive events, then $P(A) + P(B) + P(C)$ equals to

- (a) $1/3$
- (b) 1
- (c) 0
- (d) any value between 0 and 1.



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SET B

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Write down the correct answer. Each question carries 2 mark.

Que. 8 A, B and C are three mutually exclusive and exhaustive events such that $P(A) = 2P(B) = 3P(C)$. What is $P(B)$?

(a) $6/11$

(b) $3/11$

(c) $1/6$

(d) $1/3$



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SET B

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Write down the correct answer. Each question carries 2 mark.

Que. 14 If $P(\bar{A} \cup \bar{B}) = 5/6$, $P(A) = 1/2$ and $P(\bar{B}) = 2/3$, , what is $P(A \cup B)$?

- (a) $1/3$
- (b) $5/6$
- (c) $2/3$
- (d) $4/9$



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SET B

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Write down the correct answer. Each question carries 2 mark.

Que. 17 If $P(A) = a$, $P(B) = b$ and $P(A \cap B) = c$ then the expression of

$P(A' \cap B')$ in terms of a , b and c is

(a) $1 - a - b - c$

(b) $a + b - c$

(c) $1 + a - b - c$

(d) $1 - a - b + c$



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SET B

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Write down the correct answer. Each question carries 2 mark.

Que. 18 For three events A, B and C, the probability that only A occur is

- (a) $P(A)$
- (b) $P(A \cup B \cup C)$
- (c) $P(A' \cap B \cap C)$
- (d) $P(A \cap B' \cap C')$



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Write down the correct answer. Each question carries 5 mark.



Que. 7 A number is selected at random from the first 1000 natural numbers. What is the probability that the number so selected would be a multiple of 7 or 11?

(a) 0.25

(b) 0.32

(c) 0.22

(d) 0.33

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**CONDITIONAL
PROBABILITY**

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Example A pair of dice is thrown together and the sum of points of the two dice is noted to be 10. What is the probability that one of the two dice has shown the point 4?



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CONDITIONAL PROBABILITY

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

Provided $P(A) \neq 0$

- We use the notation $P(B/A)$, to be read as 'probability of the event B given that the event A has already occurred'



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CONDITIONAL PROBABILITY

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

if $P(B) \neq 0$.

- We use the notation $P(A/B)$, to be read as 'probability of the event A given that the event B has already occurred'



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CONDITIONAL PROBABILITY



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Event for which we are finding Conditional Probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Event which is occurred

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COMPOUND PROBABILITY / JOINT PROBABILITY

- The probability of occurrence of two events A and B simultaneously is known as the Compound Probability or Joint Probability of the events A and B and is denoted by $P(A \cap B)$.
- In a similar manner, the probability of simultaneous occurrence of K events A_1, A_2, \dots, A_k , is denoted by $P(A_1 \cap A_2 \cap \dots \cap A_k)$.



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JOINT PROBABILITY

**WITHOUT REPLACEMENT
(DEPENDENT EVENT)**

$$P(A \cap B) = P(A) \cdot P(B | A)$$

**WITH REPLACEMENT
(INDEPENDENT EVENT)**

$$P(A \cap B) = P(A) \cdot P(B)$$

It is used when we have to find simultaneous occurrence of two or more events

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COMPOUND PROBABILITY / JOINT PROBABILITY

DEPENDENT EVENTS

- In case of compound probability of 2 events A and B, we may face two different situations.
- In the first case, if the occurrence of one event, say B, is influenced by the occurrence of another event A, then the two events A and B are known as dependent events.



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EXAMPLE

- If a box contains 6 red , 7 black balls and 3 white balls . Find the probability of drawing three red balls without replacement .



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COMPOUND THEOREM (DEPENDENT EVENTS)

If A and B are two dependent events , then

$$P(A \cap B) = P(A) \times P(B/A)$$

Or

$$P(A \cap B) = P(B) \times P(A/B)$$



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COMPOUND THEOREM (DEPENDENT EVENTS)

If A , B and C are dependent events then ,

$$P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/(A \cap B))$$



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INDEPENDENT EVENTS

- In the second scenario, if the occurrence of the second event B is not influenced by the occurrence of the first event A, then B is known to be independent of A.
- It also follows that in this case, A is also independent of B and A and B are known as mutually independent or just independent.

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INDEPENDENT EVENTS

$$P(B/A) = P(B)$$

and also $P(A/B) = P(A)$

There by implying, $P(A \cap B) = P(A) \times P(B)$



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EXAMPLE

- If a box contains 6 red , 7 black balls and 3 white balls . Find the probability of drawing three red balls with replacement .



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COMPOUND THEOREM (INDEPENDENT EVENTS)



In case of independent events

- $P(A \cap B) = P(A) \times P(B)$
- $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$

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RESULT

- Three events A, B and C are known as independent if the following conditions hold:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap C) = P(A) \times P(C)$$

$$P(B \cap C) = P(B) \times P(C)$$

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

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RESULT

- It may be further noted that if two events A and B are independent, then the following pairs of events are also independent:

(i) A and B'

(ii) A' and B

(iii) A' and B'

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Example Rupesh is known to hit a target in 5 out of 9 shots whereas David is known to hit the same target in 6 out of 11 shots. What is the probability that the target would be hit once they both try?



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Example In a group of 20 males and 15 females, 12 males and 8 females are service holders.

What is the probability that a person selected at random from the group is a service holder given that the selected person is a male?



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Example The odds in favour of an event is 2 : 3 and the odds against another event is 3 : 7.

Find the probability that only one of the two events occurs.



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Example In connection with a random experiment, it is found that

$$P(A) = \frac{2}{3}, P(B) = \frac{3}{5} \text{ and } P(A \cup B) = \frac{5}{6}$$

Evaluate the following probabilities:

(i) $P(A/B)$

(ii) $P(B/A)$

(iii) $P(A'/B)$

(iv) $P(A/B')$

(v) $P(A'/B')$



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Example There are three boxes with the following compositions :

Colour \ Box	Blue	Red	White	Total
I	5	8	10	23
II	4	9	8	21
III	3	6	7	16

One ball is drawn from each box. What is the probability that they would be of the same colour?



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Example Mr. Roy is selected for three separate posts. For the first post, there are three candidates, for the second, there are five candidates and for the third, there are 10 candidates.

What is the probability that Mr. Roy would be selected?



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Example The independent probabilities that the three sections of a costing department will encounter a computer error are 0.2, 0.3 and 0.1 per week respectively. What is the probability that there would be

- (i) at least one computer error per week?
- (ii) one and only one computer error per week?



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Example There are two urns containing 5 red and 6 white balls and 3 red and 7 white balls respectively. If two balls are drawn from the first urn without replacement and transferred to the second urn and then a draw of another two balls is made from it, what is the probability that both the balls drawn are red?



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Example If 8 balls are distributed at random among three boxes, what is the probability that the first box would contain 3 balls?



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CODE : SS12 Write down the correct answer. Each question carries 1 mark.

Que. 12 If for two events A and B, $P(A \cap B) \neq P(A) \times P(B)$, then the two events A and B are

- (a) Independent
- (b) Dependent
- (c) Not equally likely
- (d) Not exhaustive.



CODE : SS12 Write down the correct answer. Each question carries 1 mark.



Que. 14 If two events A and B are independent, then

- (a) A and the complement of B are independent
- (b) B and the complement of A are independent
- (c) Complements of A and B are independent
- (d) All of these (a), (b) and (c).

CODE : SS12 Write down the correct answer. Each question carries 1 mark.

Que. 25 $P(B/A)$ is defined only when

- (a) A is a sure event
- (b) B is a sure event
- (c) A is not an impossible event
- (d) B is an impossible event.



CODE : SS12

Write down the correct answer. Each question carries 1 mark.

Que. 50 A, B, C are three mutually independent with probabilities 0.3, 0.2 and 0.4 respectively. What is $P(A \cap B \cap C)$?

- (a) 0.400
- (b) 0.240
- (c) 0.024
- (d) 0.500



CODE : SS12

Write down the correct answer. Each question carries 1 mark.

Que. 55 What is the probability of having at least one 'six' from 3 throws of a perfect die?

(a) $5/6$

(b) $(5/6)^3$

(c) $1 - (1/6)^3$

(d) $1 - (5/6)^3$



CODE : SS12

Write down the correct answer. Each question carries 2 mark.

Que. 5 If A, B and C are mutually exclusive independent and exhaustive events then what is the probability that they occur simultaneously?

- (a) 1
- (b) 0.50
- (c) 0
- (d) any value between 0 and 1



CODE : SS12

Write down the correct answer. Each question carries 2 mark.

Que. 9 For two events A and B, $P(B) = 0.3$, $P(A \text{ but not } B) = 0.4$ and

$P(\text{not } A) = 0.6$. The events A and B are

- (a) exhaustive
- (b) independent
- (c) equally likely
- (d) mutually exclusive



CODE : SS12

Write down the correct answer. Each question carries 2 mark.

Que. 12 For two independent events A and B, what is $P(A+B)$, given

$P(A) = 3/5$ and $P(B) = 2/3$?

- (a) $11/15$
- (b) $13/15$
- (c) $7/15$
- (d) 0.65



CODE : SS12

Write down the correct answer. Each question carries 2 mark.

Que. 11 Given that for two events A and B, $P(A) = 3/5$, $P(B) = 2/3$ and

$P(A \cup B) = 3/4$, what is $P(A/B)$?

- (a) 0.655
- (b) 13/60
- (c) 31/60
- (d) 0.775



CODE : SS12

Write down the correct answer. Each question carries 2 mark.

Que. 15 If for two independent events A and B, $P(A \cup B) = \frac{2}{3}$ and $P(A) = \frac{2}{5}$, what is $P(B)$?

- (a) $\frac{4}{15}$
- (b) $\frac{4}{9}$
- (c) $\frac{5}{9}$
- (d) $\frac{7}{15}$



CODE : SS12 Write down the correct answer. Each question carries 2 mark.

Que. 16 If $P(A) = 2/3$, $P(B) = 3/4$, $P(A/B) = 2/3$, then what is $P(B/A)$?

(a) $1/3$

(b) $2/3$

(c) $3/4$

(d) $1/2$



CODE : SS12

Write down the correct answer. Each question carries 2 mark.

Que. 19 It is given that a family of 2 children has a girl, what is the probability that the other child is also a girl ?

(a) 0.50

(b) 0.75

(c) $1/3$

(d) $2/3$



CODE : SS12

Write down the correct answer. Each question carries 2 mark.



Que. 20 Two coins are tossed simultaneously. What is the probability that the second coin would show a tail given that the first coin has shown a head?

- (a) 0.50
- (b) 0.25
- (c) 0.75
- (d) 0.125

CODE : SS12**Write down the correct answer. Each question carries 5 mark.**

Que. 5 A box contains 5 white and 7 black balls. Two successive drawn of 3 balls are made (i) with replacement (ii) without replacement. The probability that the first draw would produce white balls and the second draw would produce black balls are respectively

- (a) $6/321$ and $3/926$
- (b) $1/20$ and $1/30$
- (c) $35/144$ and $35/108$
- (d) $7/968$ and $5/264$

CODE : SS12 Write down the correct answer. Each question carries 5 mark.

Que. 6 There are three boxes with the following composition:

Box I: 5 Red + 7 White + 6 Blue balls

Box II: 4 Red + 8 White + 6 Blue balls

Box III: 3 Red + 4 White + 2 Blue balls

If one ball is drawn at random, then what is the probability that they would be of same colour?

(a) $89/729$

(b) $97/729$

(c) $82/729$

(d) $23/32$



CODE : SS12 Write down the correct answer. Each question carries 5 mark.

Que. 8 A bag contains 8 red and 5 white balls. Two successive draws of 3 balls are made without replacement. The probability that the first draw will produce 3 white balls and the second 3 red balls is

- (a) $5/223$
- (b) $6/257$
- (c) $7/429$
- (d) $3/548$



CODE : SS12 Write down the correct answer. Each question carries 5 mark.

Que. 10 A problem in probability was given to three CA students A, B and C whose chances of solving it are $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{2}$ respectively. What is the probability that the problem would be solved?

- (a) $\frac{4}{15}$
- (b) $\frac{7}{8}$
- (c) $\frac{8}{15}$
- (d) $\frac{11}{15}$



CODE : SS12 *Write down the correct answer. Each question carries 5 mark.*



Que. 11 There are three persons aged 60, 65 and 70 years old. The survival probabilities for these three persons for another 5 years are 0.7, 0.4 and 0.2 respectively. What is the probability that at least two of them would survive another five years?

- (a) 0.425
- (b) 0.456
- (c) 0.392
- (d) 0.388

CODE : SS12 Write down the correct answer. Each question carries 5 mark.

Que. 12 Tom speaks truth in 30 percent cases and Dick speaks truth in 25 percent cases. What is the probability that they would contradict each other?

- (a) 0.325
- (b) 0.400
- (c) 0.925
- (d) 0.075



CODE : SS12**Write down the correct answer. Each question carries 5 mark.**

Que. 13 There are two urns. The first urn contains 3 red and 5 white balls whereas the second urn contains 4 red and 6 white balls. A ball is taken at random from the first urn and is transferred to the second urn. Now another ball is selected at random from the second urn. The probability that the second ball would be red is

- (a) $7/20$
- (b) $35/88$
- (c) $17/52$
- (d) $3/20$

CODE : SS12 *Write down the correct answer. Each question carries 5 mark.*



Que. 14 For a group of students, 30 %, 40% and 50% failed in Physics , Chemistry and at least one of the two subjects respectively. If an examinee is selected at random, what is the probability that he passed in Physics if it is known that he failed in Chemistry?

- (a) $1/2$
- (b) $1/3$
- (c) $1/4$
- (d) $1/6$

CODE : SS12

Write down the correct answer. Each question carries 5 mark.

Que. 18 Given that $P(A) = 1/2$, $P(B) = 1/3$, $P(A \cap B) = 1/4$, what is

$P(A'/B')$

(a) $1/2$

(b) $7/8$

(c) $5/8$

(d) $2/3$



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RANDOM VARIABLE

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RANDOM VARIABLE

- A random variable or stochastic variable is a function defined on a sample space associated with a random experiment assuming any value from \mathbb{R} and assigning a real number to each and every sample point of the random experiment.
- A random variable is denoted by a capital letter.



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EXAMPLE

If a coin is tossed three times and if X denotes the number of heads , then X is a random variable variable.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$X = 0$ if the sample point is TTT

$X = 1$ if the sample point is HTT, THT or TTH

$X = 2$ if the sample point is HHT, HTH or THH

$X = 3$ if the sample point is HHH.



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TYPES

- **DISCRETE RANDOM VARIABLE :** A random variable defined on a discrete sample space is known as a discrete random variable and it can assume either only a finite number or a countably infinite number of values.
- The number of car accident, the number of heads etc. are examples of discrete random variables.

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TYPES

- **CONTINUOUS RANDOM VARIABLE** : A continuous random variable, like height, weight etc. is a random variable defined on a continuous sample space and assuming an uncountably infinite number of values.

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PROBABILITY DISTRIBUTION

- The probability distribution of a random variable may be defined as a statement expressing the different values taken by a random variable and the corresponding probabilities.
- Then if a random variable X assumes n finite values $X_1, X_2, X_3, \dots, X_n$ with corresponding probabilities $P_1, P_2, P_3, \dots, P_n$ such that



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PROBABILITY DISTRIBUTION

- (i) $p_i \geq 0$ for every i
- (ii) $\sum p_i = 1$ (over all i)
- then the probability distribution of the random variable X is given by

X:	X_1	X_2	X_3 X_n	Total
P:	P_1	P_2	P_3 P_n	1

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EXPECTED VALUE OF RANDOM VARIABLE

- Expected value or Mathematical Expectation or Expectation of a random variable may be defined as the sum of products of the different values taken by the random variable and the corresponding probabilities.
- Hence, if a random variable x assumes n values $x_1, x_2, x_3, \dots, x_n$ with corresponding probabilities $p_1, p_2, p_3, \dots, p_n$

$$\mu = E(x) = \sum p_i x_i$$

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VARIANCE OF RANDOM VARIABLE

Variance of x , to be denoted by , σ^2 is given by

$$\begin{aligned}V(x) &= \sigma^2 = E(x - \mu)^2 \\ &= E(x^2) - \mu^2\end{aligned}$$



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NOTE

- The positive square root of variance is known as standard deviation and is denoted by σ .
- If $y = a + b x$, for two random variables x and y and for a pair of constants a and b , then the mean i.e. expected value of y is given by

$$\mu_y = a + b \mu_x$$

and the standard deviation of y is

$$\sigma_y = |b| \times \sigma_x$$



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PROPERTIES OF EXPECTED VALUE

1. Expectation of a constant k is k
i.e. $E(k) = k$ for any constant k .
2. Expectation of sum of two random variables is the sum of their expectations.
i.e. $E(x + y) = E(x) + E(y)$ for any two random variables x and y .
3. Expectation of the product of a constant and a random variable is the product of the constant and the expectation of the random variable.
i.e. $E(kx) = k.E(x)$ for any constant k

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PROPERTIES OF EXPECTED VALUE

4. Expectation of the product of two random variables is the product of the expectation of the two random variables, provided the two variables are independent.

i.e. $E(xy) = E(x) \times E(y)$

Whenever x and y are independent.



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Example An unbiased coin is tossed three times. Find the expected value of the number of heads and also its standard deviation.



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Example A random variable has the following probability distribution:

X:	4	5	7	8	10
P:	0.15	0.20	0.40	0.15	0.10

Find $E [x - E(x)]^2$. Also obtain $v(3x - 4)$



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Example In a business venture, a man can make a profit of ₹ 50,000 or incur a loss of ₹ 20,000. The probabilities of making profit or incurring loss, from the past experience, are known to be 0.75 and 0.25 respectively. What is his expected profit?



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Example A box contains 12 electric lamps of which 5 are defectives. A man selects three lamps at random. What is the expected number of defective lamps in his selection?



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Example Moidul draws 2 balls from a bag containing 3 white and 5 Red balls. He gets ₹ 500 if he draws a white ball and ₹ 200 if he draws a red ball. What is his expectation? If he is asked to pay ₹ 400 for participating in the game, would he consider it a fair game and participate?



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Example A number is selected at random from a set containing the first 100 natural numbers and another number is selected at random from another set containing the first 200 natural numbers. What is the expected value of the product?



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Example A random variable x has the following probability distribution :

X	:	0	1	2	3	4	5	6	7
$P(X)$:	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$	$2k^2+k$

Find (i) the value of k

(ii) $P(x < 3)$

(iii) $P(x \geq 4)$

(iv) $P(2 < x \leq 5)$



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CODE : SS12 Write down the correct answer. Each question carries 1 mark.

Que. 38 Values of a random variable are

- (a) always positive numbers.
- (b) always positive real numbers.
- (c) real numbers.
- (d) natural numbers.



CODE : SS12 Write down the correct answer. Each question carries 1 mark.

Que. 39 Expected value of a random variable

- (a) is always positive
- (b) may be positive or negative
- (c) may be positive or negative or zero
- (d) can never be zero.



CODE : SS12 Write down the correct answer. Each question carries 1 mark.

Que. 40 If all the values taken by a random variable are equal then

- (a) its expected value is zero
- (b) its standard deviation is zero
- (c) its standard deviation is positive
- (d) its standard deviation is a real number.



CODE : SS12

Write down the correct answer. Each question carries 1 mark.

Que. 53 If x and y are random variables having expected values as 4.5 and 2.5 respectively, then the expected value of $(x-y)$ is

- (a) 2
- (b) 7
- (c) 6
- (d) 0



CODE : SS12

Write down the correct answer. Each question carries 1 mark.

Que. 54 If variance of a random variable x is 23, then what is the variance of $2x + 10$?

- (a) 56
- (b) 33
- (c) 46
- (d) 92



CODE : SS12**Write down the correct answer. Each question carries 2 mark.**

Que. 21 If a random variable x assumes the values 0, 1 and 2 with probabilities 0.30, 0.50 and 0.20, then its expected value is

(a) 1.50

(b) 3

(c) 0.90

(d) 1



CODE : SS12**Write down the correct answer. Each question carries 2 mark.****Que. 22** If two random variables x and y are related as $y = -3x + 4$ and standard deviation of x is 2, then the standard deviation of y is

- (a) - 6
- (b) 6
- (c) 18
- (d) 3.50



CODE : SS12

Write down the correct answer. Each question carries 2 mark.

Que. 23 If $2x + 3y + 4 = 0$ and $v(x) = 6$ then $v(y)$ is

(a) $8/3$

(b) 9

(c) - 9

(d) 6



CODE : SS12**Write down the correct answer. Each question carries 5 mark.****Que. 20** The probability distribution of a random variable x is given below:

$x :$	1	2	4	5	6
$P :$	0.15	0.25	0.20	0.30	0.10

What is the standard deviation of x ?

- (a) 1.49
- (b) 1.56
- (c) 1.69
- (d) 1.72

CODE : SS12 Write down the correct answer. Each question carries 5 mark.

Que. 21 A packet of 10 electronic components is known to include 3 defectives. If 4 components are selected from the packet at random, what is the expected value of the number of defective?

- (a) 1.20
- (b) 1.21
- (c) 1.69
- (d) 1.72



CODE : SS12**Write down the correct answer. Each question carries 5 mark.**

Que. 22 The probability that there is at least one error in an account statement prepared by 3 persons A, B and C are 0.2, 0.3 and 0.1 respectively. If A, B and C prepare 60, 70 and 90 such statements, then the expected number of correct statements

- (a) 170
- (b) 176
- (c) 178
- (d) 180



CODE : SS12**Write down the correct answer. Each question carries 5 mark.**

Que. 23 A bag contains 6 white and 4 red balls. If a person draws 2 balls and receives ₹ 10 and ₹ 20 for a white and red balls respectively, then his expected amount is

- (a) ₹ 25
- (b) ₹ 26
- (c) ₹ 29
- (d) ₹ 28



CODE : SS12 Write down the correct answer. Each question carries 5 mark.**Que. 24** The probability distribution of a random variable is as follows:

x :	1	2	4	6	8
P :	k	2k	3k	3k	k

The variance of x is

- (a) 2.1
- (b) 4.41
- (c) 2.32
- (d) 2.47



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**STATISTICAL
DEFINITION OF
PROBABILITY**

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Shivani
Sharma

RELATIVE FREQUENCY DEFINITION OF PROBABILITY

- Let us consider a random experiment repeated a very good number of times, say n , under an identical set of conditions. We next assume that an event A occurs f_A times. Then the limiting value of the ratio of f_A to n as n tends to infinity is defined as the probability of A .

$$\text{i.e. } P(A) = \lim_{n \rightarrow \infty} \frac{f_A}{n}$$

This statistical definition is applicable if the above limit exists and tends to a finite value.

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Example The following data relate to the distribution of wages of a group of workers:

Wages in ₹:	50-60	60-70	70-80	80-90	90-100	100-110	110-120
No. of workers:	15	23	36	42	17	12	5

If a worker is selected at random from the entire group of workers, what is the probability that

- (a) his wage would be less than ₹ 50?
- (b) his wage would be less than ₹ 80?
- (c) his wage would be more than ₹ 100?
- (d) his wages would be between ₹ 70 and ₹ 100?



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